

Please check the examination details below before entering your candidate information

Candidate surname

Other names

Pearson Edexcel
International
Advanced Level

Centre Number

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Monday 18 January 2021

Morning (Time: 1 hour 30 minutes)

Paper Reference **WMA12/01**

Mathematics

International Advanced Subsidiary/Advanced Level
Pure Mathematics P2

You must have:

Mathematical Formulae and Statistical Tables (Lilac), calculator

Total Marks

Candidates may use any calculator permitted by Pearson regulations. Calculators must not have the facility for symbolic algebra manipulation, differentiation and integration, or have retrievable mathematical formulae stored in them.

Instructions

- Use **black** ink or ball-point pen.
- If pencil is used for diagrams/sketches/graphs it must be dark (HB or B).
- **Fill in the boxes** at the top of this page with your name, centre number and candidate number.
- Answer **all** questions and ensure that your answers to parts of questions are clearly labelled.
- Answer the questions in the spaces provided
– *there may be more space than you need.*
- You should show sufficient working to make your methods clear. Answers without working may not gain full credit.
- Inexact answers should be given to three significant figures unless otherwise stated.

Information

- A booklet 'Mathematical Formulae and Statistical Tables' is provided.
- There are 10 questions in this question paper. The total mark for this paper is 75.
- The marks for **each** question are shown in brackets
– *use this as a guide as to how much time to spend on each question.*

Advice

- Read each question carefully before you start to answer it.
- Try to answer every question.
- Check your answers if you have time at the end.
- If you change your mind about an answer, cross it out and put your new answer and any working underneath.

Turn over ►

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Pearson

1.

$$f(x) = x^4 + ax^3 - 3x^2 + bx + 5$$

where a and b are constants.

When $f(x)$ is divided by $(x+1)$, the remainder is 4

(a) Show that $a + b = -1$

(2)

When $f(x)$ is divided by $(x-2)$, the remainder is -23

(b) Find the value of a and the value of b .

(4)

a. Remainder theorem: when a polynomial, $f(x)$, is divided by $(x-a)$, the remainder is equal to $f(a)$

$$x+1 = 0$$

$$x = -1$$

Subbing $x = -1$ into $f(x)$ will give us the remainder

$$f(-1) = (-1)^4 + a(-1)^3 - 3(-1)^2 + b(-1) + 5 \quad \checkmark \text{ M1}$$

$$4 = 1 - a - 3 - b + 5$$

$$4 = -a - b + 3$$

$$a + b = -1 \quad \textcircled{1}$$

$$a + b = -1 \quad \checkmark \text{ A1*}$$

b. Repeating the steps in part (a), $f(2)$ is equal to the remainder -23

$$f(2) = (2)^4 + a(2)^3 - 3(2)^2 + b(2) + 5 \quad \checkmark \text{ M1}$$

$$-23 = 16 + 8a - 12 + 2b + 5$$

$$= 8a + 2b + 9$$

$$8a + 2b = -32 \quad \checkmark \text{ A1}$$

$$4a + b = -16 \quad \textcircled{2}$$

$$\textcircled{2} - \textcircled{1}.$$

$$3a = -15$$

$$a = -5 \quad \checkmark \text{ dM1}$$

Sub $a = -5$ into $\textcircled{1}$

$$-5 + b = -1$$

$$b = 4$$

$$a = -5, b = 4 \quad \checkmark \text{ A1}$$

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Question 1 continued

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Question 1 continued

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(Total 6 marks)

Q1



2. A curve has equation

$$y = x^3 - x^2 - 16x + 2$$

- (a) Using calculus, find the x coordinates of the stationary points of the curve. (4)
- (b) Justify, by further calculus, the nature of all of the stationary points of the curve. (3)

a. Stationary points is where $\frac{dy}{dx} = 0$

$$\frac{dy}{dx} = 3x^2 - 2x - 16 \quad \checkmark M1 \quad \checkmark A1$$

$$3x^2 - 2x - 16 = 0 \quad \checkmark M1$$

$$(x+2)(3x-8) = 0$$

$$x = -2 \quad x = \frac{8}{3} \quad \checkmark A1$$

b. When $\frac{d^2y}{dx^2} > 0$, stationary point is a minimum

When $\frac{d^2y}{dx^2} < 0$, stationary point is a maximum

$$\frac{d^2y}{dx^2} = 6x - 2 \quad \checkmark M1$$

$$x = -2$$

$$x = \frac{8}{3}$$

$$\begin{aligned} \frac{d^2y}{dx^2} &= 6(-2) - 2 \\ &= -12 - 2 \\ &= -14 \end{aligned}$$

$$\begin{aligned} \frac{d^2y}{dx^2} &= 6\left(\frac{8}{3}\right) - 2 \\ &= 16 - 2 \\ &= 14 \end{aligned}$$

$-14 < 0$, $x = -2$ is a maximum $\checkmark dM1 \quad \checkmark A1$

$14 > 0$, $x = \frac{8}{3}$ is a minimum

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Question 2 continued

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Q2

(Total 7 marks)



3. (i) Solve

$$7^{x+2} = 3$$

giving your answer in the form $x = \log_7 a$ where a is a rational number in its simplest form.

(3)

(ii) Using the laws of logarithms, solve

$$1 + \log_2 y + \log_2 (y + 4) = \log_2 (5 - y)$$

(5)

i. $\log(a^b) = b \log(a)$ $\log\left(\frac{a}{b}\right) = \log(a) - \log(b)$

$$7^{x+2} = 3$$

$$\log_7(7^{x+2}) = \log_7(3)$$

$$(x+2)\log_7(7) = \log_7(3) \quad \textcircled{1} \quad \checkmark M1$$

$$x+2 = \log_7(3)$$

$$x = \log_7(3) - 2 \quad \leftarrow 2 = \log_7(49) \quad \checkmark A1$$

$$x = \log_7(3) - \log_7(49)$$

$$x = \log_7\left(\frac{3}{49}\right) \quad \textcircled{2} \quad \checkmark A1$$

③

ii. $\log(ab) = \log a + \log b$

$$1 + \log_2(y) + \log_2(y+4) = \log_2(5-y)$$

$$1 + \log_2[y(y+4)] = \log_2(5-y) \quad \textcircled{3} \quad \checkmark M1$$

$$\log_2[y(y+4)] - \log_2(5-y) = -1$$

$$\log_2\left[\frac{y(y+4)}{5-y}\right] = -1 \quad \textcircled{2}$$

$$\frac{y(y+4)}{5-y} = 2^{-1} \quad \checkmark dm1$$

$$y(y+4) = \frac{5}{2} - \frac{1}{2}y$$

$$\times 2 \quad y^2 + 4y + \frac{1}{2}y - \frac{5}{2} = 0$$

$$2y^2 + 8y + y - 5 = 0$$

$$2y^2 + 9y - 5 = 0 \quad \checkmark A1$$

$$(2y-1)(y+5) = 0 \quad \checkmark ddM1$$

$$2y-1=0 \quad y+5=0 \quad \therefore \text{Has to be } y=\frac{1}{2} \text{ as } \log_2(-5+4) = \log_2(-1), \text{ not possible to log -ve number}$$

$$y = \frac{1}{2} \quad y = -5 \quad y = \frac{1}{2} \quad \checkmark A1$$

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Question 3 continued

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(Total 8 marks)

Q3



4. (a) Find the first three terms, in ascending powers of x , of the binomial expansion of

$$(2 + px)^6$$

where p is a constant. Give each term in simplest form.

(4)

Given that in the expansion of

$$\left(3 - \frac{1}{2}x\right)(2 + px)^6$$

the coefficient of x^2 is $-\frac{3}{4}$

- (b) find the possible values of p .

(4)

a. BINOMIAL EXPANSION FORMULA:

$$(a + bx)^n = a^n + \binom{n}{1}a^{n-1}(bx)^1 + \binom{n}{2}a^{n-2}(bx)^2 + \dots + \binom{n}{n-1}a^1(bx)^{n-1} + (bx)^n$$

✓ This is the CHOOSE function on your calculator

$$\begin{aligned} (2 + px)^6 &= 2^6 + \binom{6}{1}2^5(px)^1 + \binom{6}{2}2^4(px)^2 \\ &= 64 + 6 \times 32 \times px + 15 \times 16 \times p^2x^2 \quad \checkmark M1 \quad \text{First 3 terms needed only} \\ &= 64 + 192px + 240p^2x^2 \\ &= 64 + 192px + 240p^2x^2 \end{aligned}$$

$$b. \left(3 - \frac{1}{2}x\right)(2 + px)^6 = \left(3 - \frac{1}{2}x\right)(64 + 192px + 240p^2x^2)$$

These would give the x^2 terms

$$-\frac{3}{4}x^2 = 3 \times 240p^2x^2 + \left(-\frac{1}{2}x\right) \times 192px \quad \checkmark M1$$

$$-\frac{3}{4}x^2 = 720p^2x^2 - 96px^2$$

$$-\frac{3}{4}x^2 = (720p^2 - 96p)x^2$$

Comparing coefficients

$$-\frac{3}{4} = 720p^2 - 96p \quad \checkmark dM1$$

$$\times 4 \quad 720p^2 - 96p + \frac{3}{4} = 0$$

$$2880p^2 - 384p + 3 = 0 \quad \checkmark ddM1$$

QUADRATIC FORMULA

$$p = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$p = \frac{-(-384) \pm \sqrt{(-384)^2 - 4(2880)(3)}}{2(2880)}$$



Question 4 continued

$$p = \frac{1}{8}, \quad p = \frac{1}{120} \quad \checkmark A1$$

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Question 4 continued

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Question 4 continued

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(Total 8 marks)

Q4



5. (i) Use algebra to prove that for all $x \geq 0$

$$3x + 1 \geq 2\sqrt{3x}$$

(3)

- (ii) Show that the following statement is not true.

“The sum of three consecutive prime numbers is always a multiple of 5”

(1)

- i. For these types of proofs, we can use the statement we need to prove to derive a true statement, then work backwards from there to prove the original statement

Original statement:

$$3x+1 \geq 2\sqrt{3x} \quad \text{square both sides}$$

$$(3x+1)^2 \geq (2\sqrt{3x})^2 \quad \checkmark M1$$

$$9x^2 + 6x + 1 \geq 4 \times 3x$$

$$9x^2 + 6x + 1 \geq 12x$$

$$9x^2 - 6x + 1 \geq 0 \quad \text{factorise}$$

$$(3x-1)^2 \geq 0 \quad \checkmark M1$$

This is a true statement as all square numbers cannot be negative

Actual proof:

$$(3x-1)^2 \geq 0$$

$$9x^2 - 6x + 1 \geq 0$$

$$9x^2 + 6x + 1 \geq 12x$$

$$(3x+1)^2 \geq 4 \times 3x$$

$$(3x+1)^2 \geq (2\sqrt{3x})^2$$

$$3x+1 \geq 2\sqrt{3x}$$

This part is not required in the MS, but it clearly shows how $(3x-1)^2 \geq 0$ is related to $3x+1 \geq 2\sqrt{3x}$

Square numbers are greater than or equal to 0 so $(3x-1)^2 \geq 0$ is true hence $3x+1 \geq 2\sqrt{3x}$ $\checkmark A1^*$

- ii. Use a counterexample to disprove the statement

$$\text{e.g. } 5+7+11=23$$

consecutive primes

$5+7+11=23$, not divisible by 5 so statement is false $\checkmark B1$

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Question 5 continued

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(Total 4 marks)

Q5



6. (a) Show that the equation

$$\frac{3\sin\theta\cos\theta}{2\sin\theta-1} = 5\tan\theta \quad \sin\theta \neq \frac{1}{2}$$

can be written in the form

$$3\sin^3\theta + 10\sin^2\theta - 8\sin\theta = 0$$

(4)

- (b) Hence solve, for
- $-\frac{\pi}{4} < x < \frac{\pi}{4}$

$$\frac{3\sin 2x \cos 2x}{2\sin 2x - 1} = 5\tan 2x$$

giving your answers to 3 decimal places where appropriate.

(4)

$$\text{a. } \begin{array}{|l|l|} \hline \textcircled{1} & \textcircled{2} \\ \hline \tan\theta = \frac{\sin\theta}{\cos\theta} & \cos^2\theta = 1 - \sin^2\theta \\ \hline \end{array}$$

$$\frac{3\sin\theta\cos\theta}{2\sin\theta-1} = 5\tan\theta$$

$$\frac{3\sin\theta\cos\theta}{2\sin\theta-1} = \frac{5\sin\theta}{\cos\theta} \quad \textcircled{1} \quad \checkmark \text{ M1}$$

cross multiply

$$3\sin\theta\cos^2\theta = 5\sin\theta(2\sin\theta-1) \quad \checkmark \text{ M1}$$

$$3\sin\theta\cos^2\theta = 10\sin^2\theta - 5\sin\theta$$

$$3\sin\theta(1-\sin^2\theta) = 10\sin^2\theta - 5\sin\theta \quad \textcircled{2} \quad \checkmark \text{ M1}$$

$$3\sin\theta - 3\sin^3\theta = 10\sin^2\theta - 5\sin\theta$$

$$3\sin^3\theta + 10\sin^2\theta - 8\sin\theta = 0$$

$$3\sin^3\theta + 10\sin^2\theta - 8\sin\theta = 0 \quad \checkmark \text{ A1}^*$$

- b. Using the answer in part (a):
- $3\sin^3\theta + 10\sin^2\theta - 8\sin\theta = 0$

$$\frac{3\sin 2x \cos 2x}{2\sin 2x - 1} = 5\tan 2x \Rightarrow 3\sin^3 2x + 10\sin^2 2x - 8\sin 2x = 0$$

$$3\sin^3 2x + 10\sin^2 2x - 8\sin 2x = 0$$

$$\sin 2x (3\sin^2 2x + 10\sin 2x - 8) = 0 \quad \checkmark \text{ M1}$$

$$\sin 2x (3\sin 2x - 2)(\sin 2x + 4) = 0$$

Don't forget about the range of values of x !

$$-\frac{\pi}{4} < x < \frac{\pi}{4} \Rightarrow -\frac{\pi}{2} < 2x < \frac{\pi}{2}$$



Question 6 continued

$\sin 2x = 0$

$2x = 0$

$x = 0$

$3\sin 2x - 2 = 0$

$\sin 2x = \frac{2}{3} \quad \checkmark A1$

$2x = 0.7297\dots$

$x = 0.36486\dots$

$\sin 2x + 4 = 0$

$\sin 2x = -4$

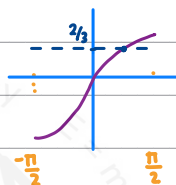
not possible as sin has range -1 to 1

$x = 0, x = 0.365 \text{ (3dp)}$

$\checkmark 81$

$\checkmark A1$

Normally you would use the CAST diagram to obtain multiple solutions, but the range of $2x$ is $-\frac{\pi}{2} < 2x < \frac{\pi}{2}$ and $\sin 2x$ is positive, so only one solution here and CAST diagram isn't needed



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Question 6 continued

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Question 6 continued

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- $\sin(x + y) = \sin x \cos y + \cos x \sin y$
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- $x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$
- $(a+b)^2 = a^2 + 2ab + b^2$
- π
- $\sqrt{x}$
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- $\frac{1}{a}$

(Total 8 marks)

Q6



7.

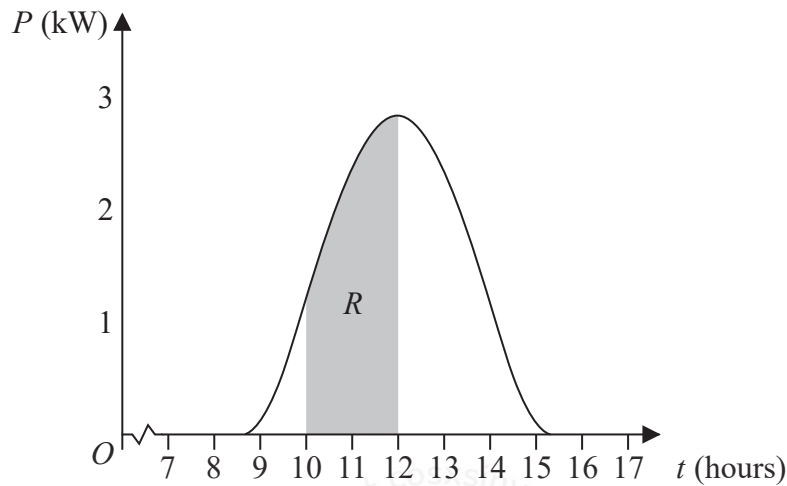


Figure 1

Solar panels are installed on the roof of a building.

The power, P , produced on a particular day, in kW, can be modelled by the equation

$$P = 0.95 + 2^{t-12} + 2^{12-t} - (t-12)^2 \quad 8.5 \leq t \leq 15.2$$

where t is the time in hours after midnight. The graph of P against t is shown in Figure 1.

A table of values of t and P is shown below, with the values of P given to 4 significant figures where appropriate.

Time, t (hours)	10	10.5	11	11.5	12
Power, P (kW)	1.2 ✓ _{B1}	1.882	2.45	2.821 ✓ _{B1}	2.95

- (a) Use the given equation to complete the table, giving the values of P to 4 significant figures where appropriate.

(2)

The amount of energy, in kWh, produced between 10:00 and 12:00 can be found by calculating the area of region R , shown shaded in Figure 1.

- (b) Use the trapezium rule, with all the values of P in the completed table, to find an estimate for the amount of energy produced between 10:00 and 12:00. Give your answer to 2 decimal places.

(4)

a. $P = 0.95 + 2^{t-12} + 2^{12-t} - (t-12)^2$

$t = 10$

$t = 11.5$

$P = 0.95 + 2^{t-10} + 2^{12-10} - (10-12)^2$

$P = 0.95 + 2^{t-11.5} + 2^{12-11.5} - (11.5-12)^2$

$P = 1.2$

$P = 2.821$ (4sf)

1.2

2.821



Question 7 continued

b. TRAPEZIUM RULE

$$\int_a^b y \, dx = \frac{1}{2} h \left[(y_0 + y_n) + 2(y_1 + y_2 + \dots + y_{n-1}) \right], \text{ where } h = \frac{b-a}{n}$$

$\uparrow$ represents the width of each strip
 $\uparrow$ heights of strips in between the first and last strip respectively
 $\uparrow$ height of the first and last strip respectively
 $\uparrow$ number of strips

$$h = \frac{12 - 10}{4}$$

4 $\uparrow$ There are 5 coordinates, so $5-1=4$ strips used

$$h = 0.5 \quad \checkmark B1$$

$$\text{energy} = \frac{1}{2} (0.5) \left[(2.95 + 1.2) + 2(1.882 + 2.45 + 2.821) \right] \quad \checkmark M1 \quad \checkmark A1ft$$

$$= 4.61 \text{ kWh (2dp)} \quad \checkmark A1$$

Q7

(Total 6 marks)



8. A sequence $a_1, a_2, a_3, \dots$ is defined by

$$a_{n+1} = 2(a_n + 3)^2 - 7$$

$$a_1 = p - 3$$

where p is a constant.

- (a) Find an expression for a_2 in terms of p , giving your answer in simplest form. (1)

Given that $\sum_{n=1}^3 a_n = p + 15$

- (b) find the possible values of a_2 (6)

a. $a_1 = p - 3$

$$a_2 = 2(a_1 + 3)^2 - 7$$

$$a_2 = 2[(p-3)+3]^2 - 7$$

$$a_2 = 2p^2 - 7$$

$$a_2 = 2p^2 - 7 \quad \checkmark B1$$

b. $\sum_{n=1}^3 a_n = p + 15$ Means that sum of first 3 terms is equal to $p + 15$

$$a_1 = p - 3 \quad a_2 = 2p^2 - 7 \quad a_3 = 2(a_2 + 3)^2 - 7$$

$$= 2[(2p^2 - 7) + 3]^2 - 7 \quad \checkmark M1$$

$$= 2(2p^2 - 4)^2 - 7$$

$$= 2(4p^4 - 16p^2 + 16) - 7$$

$$= 8p^4 - 32p^2 + 32 - 7$$

$$a_3 = 8p^4 - 32p^2 + 25$$

$$a_1 + a_2 + a_3 = p + 15$$

$$p - 3 + 2p^2 - 7 + 8p^4 - 32p^2 + 25 = p + 15 \quad \checkmark M1$$

$$8p^4 - 30p^2 + p + 15 = p + 15$$

$$8p^4 - 30p^2 = 0 \quad \checkmark A1$$

$$p^2(8p^2 - 30) = 0$$

$$p^2 = 0 \quad 8p^2 - 30 = 0$$

$$p = 0$$

$$p = \pm \sqrt{\frac{15}{4}} \quad \checkmark A1$$

$$a_2 = 2p^2 - 7$$

$$a_2 = 2(0) - 7$$

$$a_2 = 2\left(\sqrt{\frac{15}{4}}\right)^2 - 7$$

$$a_2 = -7$$

$$a_2 = \frac{1}{2}$$

$$a_2 = -7 \text{ and } a_2 = \frac{1}{2} \quad \checkmark A1$$

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Question 8 continued

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Question 8 continued

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Question 8 continued

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(Total 7 marks)

Q8



9. A circle C has equation

$$(x - k)^2 + (y - 2k)^2 = k + 7$$

where k is a positive constant.

- (a) Write down, in terms of k ,

- (i) the coordinates of the centre of C ,
(ii) the radius of C .

(2)

Given that the point $P(2, 3)$ lies on C

- (b) (i) show that $5k^2 - 17k + 6 = 0$

- (ii) hence find the possible values of k .

(3)

The tangent to the circle at P intersects the x -axis at point T .

Given that $k < 2$

- (c) calculate the exact area of triangle OPT .

(5)

a. i. centre $(k, 2k)$ ✓ B1

ii. radius $\sqrt{k+7}$ ✓ B1

b. i. Subbing in $P(2, 3)$ into $(x-k)^2 + (y-2k)^2 = k+7$

$$(2-k)^2 + (3-2k)^2 = k+7$$

$$4-4k+k^2+9-12k+4k^2 = k+7 \quad \checkmark M1$$

$$5k^2 - 17k + 6 = 0$$

$$5k^2 - 17k + 6 = 0 \quad \checkmark A1^2$$

ii. Solving for k in $5k^2 - 17k + 6 = 0$

$$\text{QUADRATIC FORMULA}$$

$$k = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$k = \frac{-(-17) \pm \sqrt{(-17)^2 - 4(5)(6)}}{2(5)}$$

$$k = \frac{2}{5}, k = 3 \quad \checkmark B1$$

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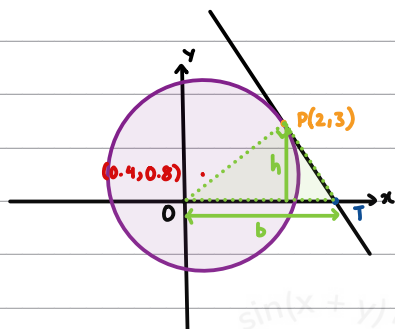


Question 9 continued

c. Since $k < 2$, $k = \frac{2}{5}$ from part (bii)

$$\text{Circle C: } (x - \frac{2}{5})^2 + (y - 2 \times \frac{2}{5})^2 = \frac{2}{5} + 7$$

$$(x - \frac{2}{5})^2 + (y - \frac{4}{5})^2 = \frac{37}{5} \Rightarrow \text{centre is } (\frac{2}{5}, \frac{4}{5}) \quad \checkmark \text{ M1ft}$$



To find area of $\triangle OPT$, we can find its base and height, but for the base, we first need to find T by finding the equation of the tangent.

POINT - GRADIENT EQUATION OF LINE

$$y - y_1 = m(x - x_1)$$

$$\text{Gradient of the radius at P: } m_P = \frac{3 - \frac{4}{5}}{2 - \frac{2}{5}} = \frac{11}{8}$$

Since radius is perpendicular to tangent.

$$m_{\text{tangent}} = -\frac{8}{11} \quad (\text{negative reciprocal of } \frac{11}{8}) \quad \checkmark \text{ M1}$$

Equation of tangent:

$$y - 3 = -\frac{8}{11}(x - 2)$$

Point T has y-coordinate zero:

$$0 - 3 = -\frac{8}{11}(x - 2) \quad \checkmark \text{ M1}$$

$$\frac{33}{8} = x - 2$$

$$x = \frac{49}{8}$$

$$T(\frac{49}{8}, 0)$$



Question 9 continued

$$\therefore \text{base} = \frac{49}{8}, \text{height} = 3$$

$$\therefore \text{Area of } \triangle OPT = \frac{1}{2}bh = \frac{1}{2} \times \frac{49}{8} \times 3 \quad \checkmark \text{dM1}$$

$$= \frac{147}{16} \quad \checkmark \text{A1}$$

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Question 9 continued

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Q9

(Total 10 marks)



10. In this question you must show detailed reasoning.

Owen wants to train for 12 weeks in preparation for running a marathon.

During the 12-week period he will run every Sunday and every Wednesday.

- On Sunday in week 1 he will run 15 km
- On Sunday in week 12 he will run 37 km

He considers two different 12-week training plans.

In training plan A, he will increase the distance he runs each Sunday by the same amount.

- (a) Calculate the distance he will run on Sunday in week 5 under training plan A. (3)

In training plan B, he will increase the distance he runs each Sunday by the same percentage.

- (b) Calculate the distance he will run on Sunday in week 5 under training plan B. Give your answer in km to one decimal place. (3)

Owen will also run a fixed distance, x km, each Wednesday over the 12-week period.

Given that

- x is an integer
 - the total distance that Owen will run on Sundays and Wednesdays over the 12 weeks will not exceed 360 km
- (c) (i) find the maximum value of x , if he uses training plan A,
(ii) find the maximum value of x , if he uses training plan B. (5)

a "By the same amount" indicates there is a common difference, which means we are dealing with an arithmetic sequence

NTH TERM OF ARITHMETIC SEQUENCE

$$a_n = a + (n-1)d$$

First term is week 1:

$$a_1 = 15$$

Week 5 / 5th term

$$a_5 = 15 + (5-1) \times 2 \quad \checkmark M1$$

$$a_5 = 23$$

Week 12 / 12th term

$$37 = 15 + (12-1)d$$

$$37 = 15 + 11d$$

$$d = 2 \quad \checkmark M1$$

$$\text{Week 5 run: } 23 \text{ km} \quad \checkmark A1$$



Question 10 continued

- b. "Same percentage" → common ratio → geometric sequence

NTH TERM OF GEOMETRIC SEQUENCE

$$a_n = ar^{n-1}$$

Week 1 / 1st term

$$a_1 = 15$$

Week 5 / 5th term

$$a_5 = 15 \left(\sqrt[11]{\frac{37}{15}} \right)^{5-1} \quad \checkmark M1$$

$$a_5 = 20.8 \text{ km (1dp)} \quad \checkmark A1$$

Week 12 / 12th term

$$37 = 15r^{12-1}$$

$$\frac{37}{15} = r^{11}$$

$$r = \sqrt[11]{\frac{37}{15}} \quad \checkmark M1$$

- c.i. For both parts of part (c), we need to solve the inequality:

sum of arithmetic/geometric series + $12x \leq 360$

ARITHMETIC SERIES

$$S_n = \frac{n}{2}(2a + (n-1)d)$$

↑ There are 12 Wednesdays

Sum of km's on Sundays

$$S_{12} = \frac{12}{2}(2 \times 15 + (12-1) \times 2)$$

$$S_{12} = 312 \quad \checkmark M1$$

$$312 + 12x \leq 360 \quad \checkmark dM1$$

$$12x \leq 48$$

$$x \leq 4$$

$$\therefore \text{Training plan A: 4 km} \quad \checkmark A1$$

ii.

GEOMETRIC SERIES

$$S_n = \frac{a(1-r^n)}{1-r}$$

$$294.185 + 12x \leq 360$$

$$12x \leq 65.815$$

$$x \leq 5.484583333$$

$$S_{12} = \frac{15(1 - (\sqrt[11]{\frac{37}{15}})^{12})}{1 - \sqrt[11]{\frac{37}{15}}} \quad \checkmark M1$$

$$\therefore \text{Training plan B: 5 km} \quad \checkmark A1 \text{ (so only if you also have 4 km)}$$

$$S_{12} = 294.185...$$



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Question 10 continued

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Q10

(Total 11 marks)

TOTAL FOR PAPER IS 75 MARKS

END

